Progress Towards the First Measurement of Direct CP-Violation in $K \to \pi\pi$ Decays From First Principles

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Outline

- **Introduction**
- $K \to (\pi\pi)_{I=2}$ **calculation.**
- $K \to (\pi\pi)_{I=0}$ **calculation.**
- **Conclusions and Outlook**

Introduction

CP-Violation in the Standard Model

- Standard Model allows violation of CP via complex phase δ in the CKM matrix.

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- Manifests in 2 ways: Direct and Indirect
- Indirect CPV arises because weak eigenstates ≠ CP eigenstates: e.g. $K_S \propto K_1 + \bar{\epsilon} K_2$ where K_1 and K_2 are CP-even and CP-odd resp.
- Also direct CPV in decays of CP eigenstates:

$$K_1 \ (\mathrm{CP-even}) \ \rightarrow \pi\pi\pi \ (\mathrm{CP-odd})$$

$$K_2 \ (\mathrm{CP-odd}) \ \rightarrow \pi\pi \ (\mathrm{CP-even})$$

Brief interlude: lattice methods

- Discretize QCD Lagrangian in Euclidean space on finite volume.
- Integrate fermions out of path integral:

$$Z = \int \mathrm{d}U \; \det(D[U]) \exp\left(-S_g[U]\right)$$

- U are gauge links: $U_\mu = e^{iaA_\mu^a T^a} \in \mathrm{SU}(3)$
- Sample configurations of links from probability distribution Z using Monte Carlo methods.

Lattice measurements

- Measure amplitudes on each link configuration and average.

$$\int \mathrm{d}^3\vec{x}\ \langle 0|\bar{d}(x)\gamma^5 u(x)\bar{u}(0)\gamma^5 d(0)|0\rangle$$
$$= \frac{1}{N}\sum_{i=1}^{N}\int \mathrm{d}^3\vec{x}\ \mathrm{tr}\left(\gamma^5 D_d^{-1}(0,x)[U_i]\gamma^5 D_u^{-1}(x,0)[U_i]\right)$$
$$= a_0 e^{-m_\pi x_4} + a_1 e^{-E_1 x_4} + \ldots$$

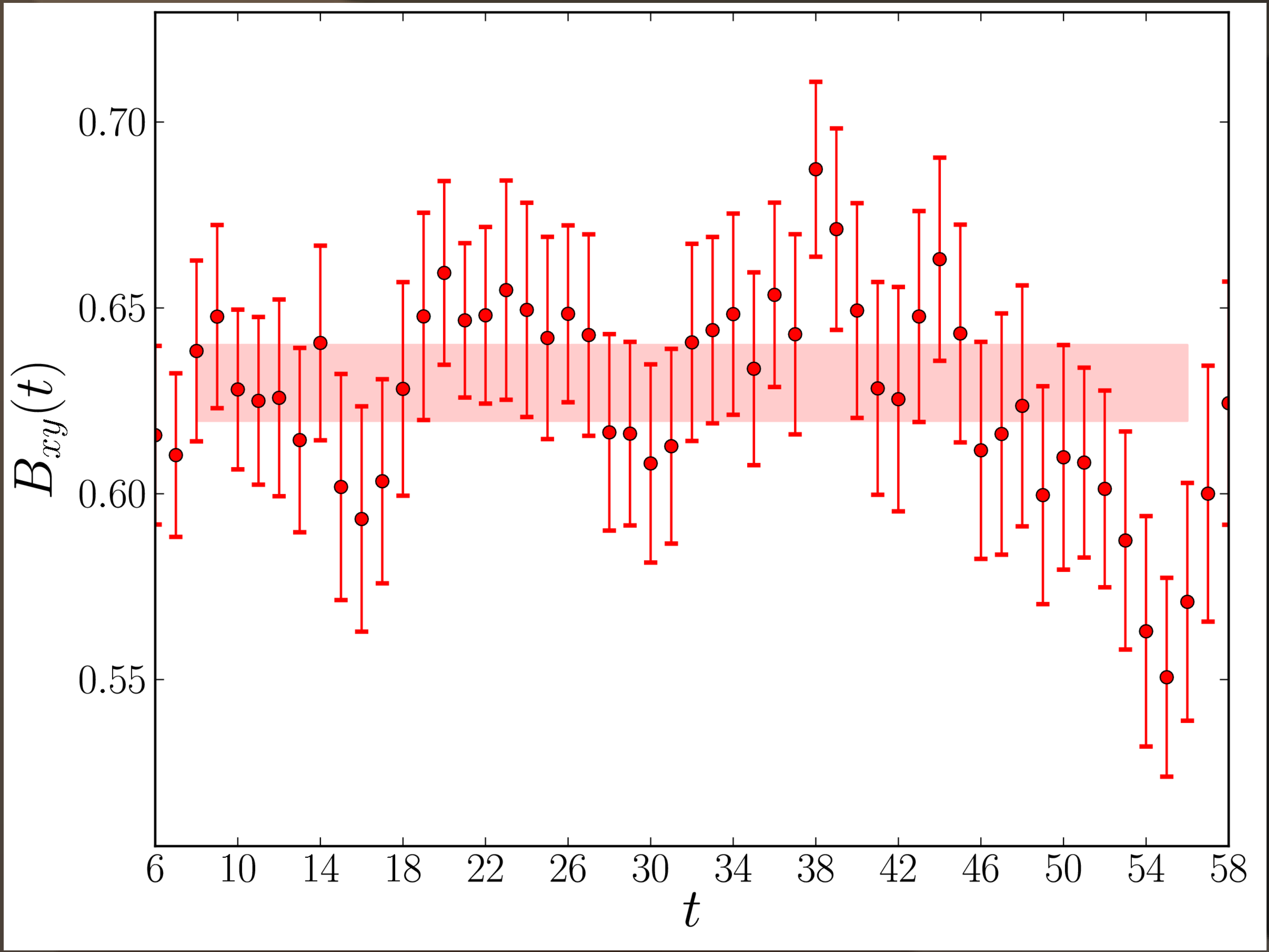
- Ground state of system extracted in limit of large time separation.
- Excited state with energy $E_i\ (i>0)$ requires multi-exponential fits to time dependence – typically very noisy and should be avoided if possible!

Indirect CP-Violation on the Lattice

- Indirect CPV measure ϵ determined accurately from experiments: $\epsilon = \dfrac{A(K_L \to (\pi\pi)_{I=0})}{A(K_S \to (\pi\pi)_{I=0})}$
- Theoretically $\epsilon \propto G_F^2 M_W^2 B_K(\mu) S(\mu)$ where $S(\mu)$ are perturbative Wilson coefficients and $B_K(\mu)$ contains the non-perturbative QCD contribution.
- Both factors are renormalization scheme dependent but their product is scheme invariant.
- On lattice we can measure B_K through

$$B_K \propto \langle \bar{K}^0 | \mathcal{H}_{\text{eff}}(\Delta S = 2) | K^0 \rangle$$

- Modern calculations at %-scale accuracy.



$B_{xy}(t)$
0.70
0.65
0.60
0.55
6
10
14
18
22
26
30
34
38
42
46
50
54
58
t

$\mathbf{K} \to \pi\pi$ *Decays*

- Direct CP-violation first observed in $K \to \pi\pi$ decays.
- Two types of decay:

$$\Delta I = 3/2 \; : K^+ \to (\pi^+\pi^0)_{I=2}$$ with amplitude A_2

$$\Delta I = 1/2 \; : K^0 \to (\pi^+\pi^-)_{I=0}$$
$$K^0 \to (\pi^0\pi^0)_{I=0}$$ with amplitude A_0

- Direct CP-violation: $$\epsilon' = \frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}}\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)$$
  where
  $\omega = \mathrm{Re}A_2/\mathrm{Re}A_0$ and δ_I are strong scattering phase shifts.
- ϵ' is highly sensitive to BSM sources of CPV.
- Strong interactions very important – origin of the so-called $\Delta I = 1/2$ rule: preference to decay to $I = 0$ final state.

K $\to \pi\pi$ *on the lattice*

- Multi-particle states in a finite box very different from infinite-volume states:

$$|\pi\pi\rangle_{\mathrm{latt}} = c_0|\pi\pi\ (l=0)\rangle_{\mathrm{phys}} + c_4|\pi\pi\ (l=4)\rangle_{\mathrm{phys}} + \ldots$$

- Until recently not known how to relate lattice amplitude to physical amplitude. [Lellouch&Luscher]
- Energy spectrum is volume-dependent; need large physical volume for realistic kinematics.
- Also need small lattice spacing to avoid large discretization errors.
- Large volume + small lattice spacing = expensive!
- Only recently become viable.

$K \to (\pi\pi)_{I=2}$ ***Calculation***

Lattice Determination

- As with B_K, amplitude A_2 is combination of renormalization-scheme dependent perturbative Wilson coeffs $C_i(\mu)$ and non-perturbative matrix elements $M_i(\mu)$:

$$A_2 \propto G_F V_{ud} V_{us} \sum_{i=1}^{10} C_i(\mu) M_i(\mu)$$

- $M_i = \langle (\pi^+ \pi^0)_{I=2} | Q_i | K^0 \rangle$
- Q_i are weak effective four-quark operators.
- Renormalization performed non-perturbatively in intermediate regularization-independent momentum scheme (RI-MOM), matched to $\overline{\mathrm{MS}}$ at high energies to avoid perturbative truncation errors.

Achieving Physical Kinematics

- $m_\pi = 135\ \mathrm{MeV}$ and $m_K = 500\ \mathrm{MeV}$: need moving pions in final state to conserve energy.
- Ground state of $\pi\pi$ system has stationary pions.
- As previously mentioned, extracting excited states is very hard. Can we avoid this? Yes!

Physical Kinematics

- Instead impose antiperiodic BCs on d-quark propagator. Changes finite-volume momentum discretization:

$$p = \frac{2\pi n}{L} \rightarrow \frac{(2n+1)\pi}{L}$$

$\pi^{\pm}$

- Minimum d-quark momentum is π/L : charged pion ground state has momentum! But...
- For neutral pion the momenta can cancel, s.t. ground state is stationary. Desired state is $\pi^{+}\pi^{0}$, so this does not work. However....
- Wigner-Eckart theorem:

$$\langle(\pi^{+}\pi^{0})_{I=2}|Q^{\Delta I_z=1/2}|K^{+}\rangle = \frac{\sqrt{3}}{2}\langle(\pi^{+}\pi^{+})_{I=2}|Q^{\Delta I_z=3/2}|K^{+}\rangle$$

- APBCs on d-quark break isospin symmetry allowing mixing between isospin states: however $\pi^{+}\pi^{+}$ is the only charge-2 state hence it cannot mix.

Results

- RBC & UKQCD recently published (arXiv:1111.1699) calculation of $\Delta I = 1/2$ decay using:
  - 2+1f domain wall fermions on a $32^3 \times 64 \times 32$ lattice with $a^{-1} = 1.37(1)\ \mathrm{GeV}.$
  - Near physical pions: $m_\pi^{PQ} \sim 140\ \mathrm{MeV}, m_\pi^{\mathrm{uni}} \sim 170\ \mathrm{MeV}$
  - Energy conserving decays
- Determined

$$\mathrm{Re}A_2 = [1.436(62)_{\mathrm{stat}}(258)_{\mathrm{sys}}] \times 10^{-8}\ \mathrm{GeV}$$

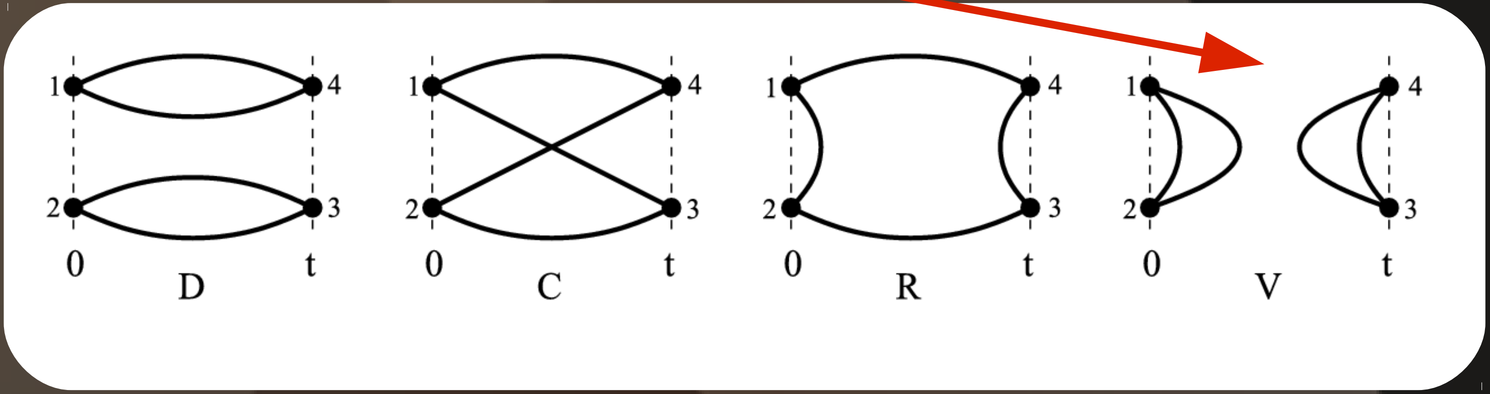
$$\mathrm{Im}A_2 = -[6.83(51)_{\mathrm{stat}}(1.30)_{\mathrm{sys}}] \times 10^{-8}\ \mathrm{GeV}$$

- Large systematic error of which 75% is discretization error: continuum limit needed.
- Currently generating multiple larger, finer lattices to get better control of this error.

$K \to (\pi\pi)_{I=0}$ ***Calculation***

Challenges: part 1

- Measuring A_0 is considerably more challenging.
- Measure both $K^0 \to \pi^+\pi^-$ and $K^0 \to \pi^0\pi^0$.
- $\pi\pi$ state has vacuum quantum numbers, hence there are disconnected diagrams:



- Need large statistics and many source positions (or A2A/AMA propagators) to resolve.
- With Blue Gene/Q resources we can now perform such calculations with large enough physical volumes.

Challenges: part 2

- For $\Delta I = 1/2$ the Wigner-Eckart trick cannot be used.
- If we stay with APBC on d-quarks, isospin-breaking would allow mixing between $I = 0$ and $I = 2$ final states.
- I=0 state needs moving π^0, but momentum cancels in $d\bar{d}$.
- Need to apply BCs that commute with isospin and produce moving π^0 as well as π^+ and π^-.
- Can we conceive boundary conditions that satisfy these criteria? Yes: G-parity.

G-Parity Boundary Conditions

- G-parity is a charge conjugation followed by a 180 degree isospin rotation about the y-axis:

Wiese, Nucl.Phys.B375, (1992)

Kim, arXiv:hep-lat/0311003 (2003)

$$\hat{G} = \hat{C} e^{i\pi \hat{I}_y} \quad : \quad \begin{aligned} \hat{G}|\pi^\pm\rangle &= -|\pi^\pm\rangle \\ \hat{G}|\pi^0\rangle &= -|\pi^0\rangle \end{aligned}$$

- Pions are all eigenstates with e-val -1, hence G-parity BCs make pion wavefunctions antiperiodic, with minimum momentum π/L.
- G-parity commutes with isospin

Kaons

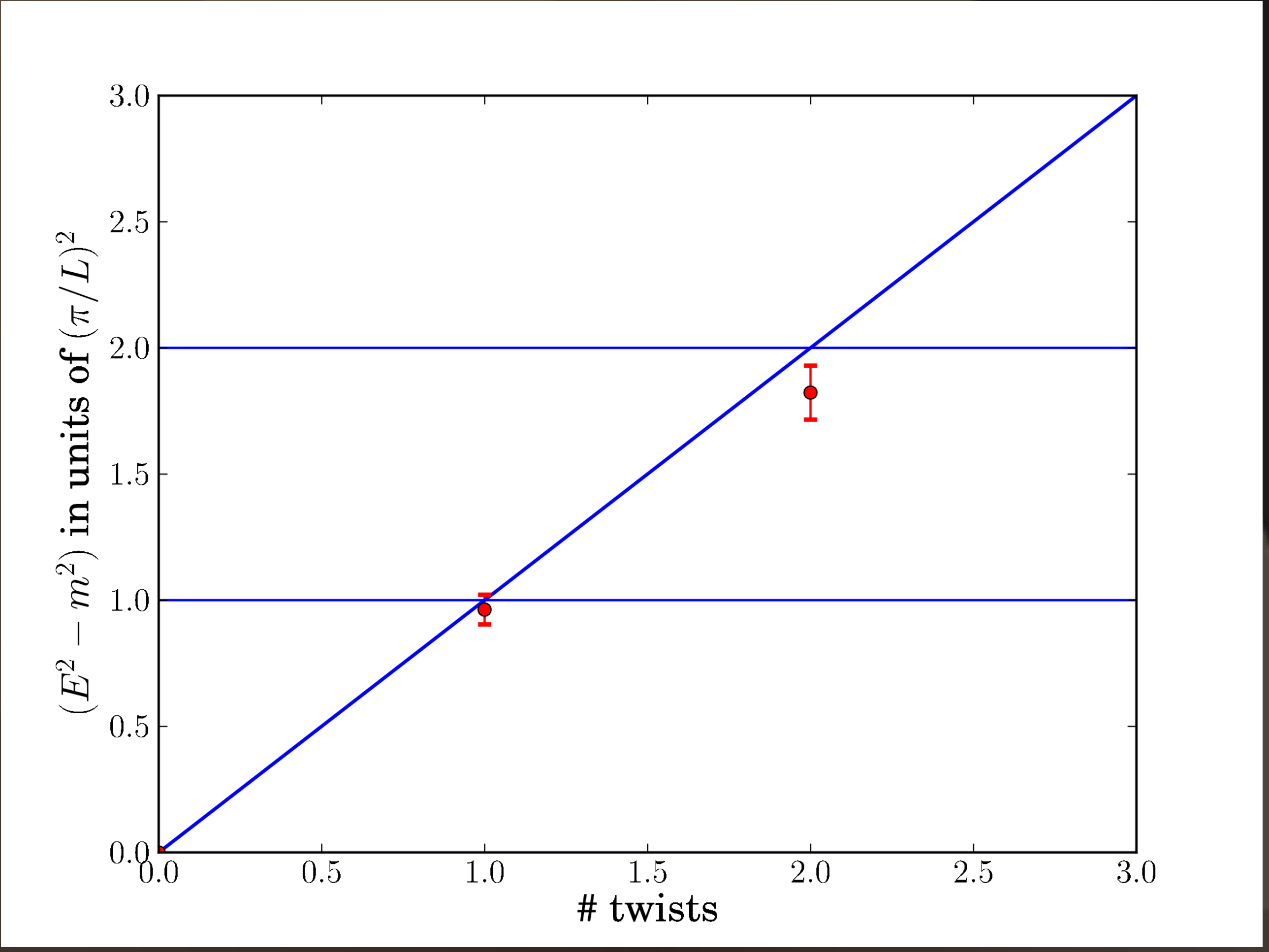
- $K \to \pi\pi$ calculation needs stationary K^0.
- Need an eigenstate with e-val $+1$ for periodic BCs and hence $p_{\rm min} = 0$.
- $\frac{1}{\sqrt{2}}(\bar{s}d + \bar{d}s)$ is not a G-parity eigenstate.
- Introduce 'strange isospin' (I'): s-quark in doublet $\left(\begin{array}{c} s' \\ s \end{array}\right)$
- A neutral kaon-like state:

  $K_0' = \frac{1}{2}(\bar{s}d + \bar{d}s + \bar{s}'u + \bar{u}s')$

  is an eigenstate of 'modified G-parity': $\hat{G} = \hat{C}e^{i\pi\hat{I}_y}e^{i\pi\hat{I}_y'}$ with e-val $+1$.   .

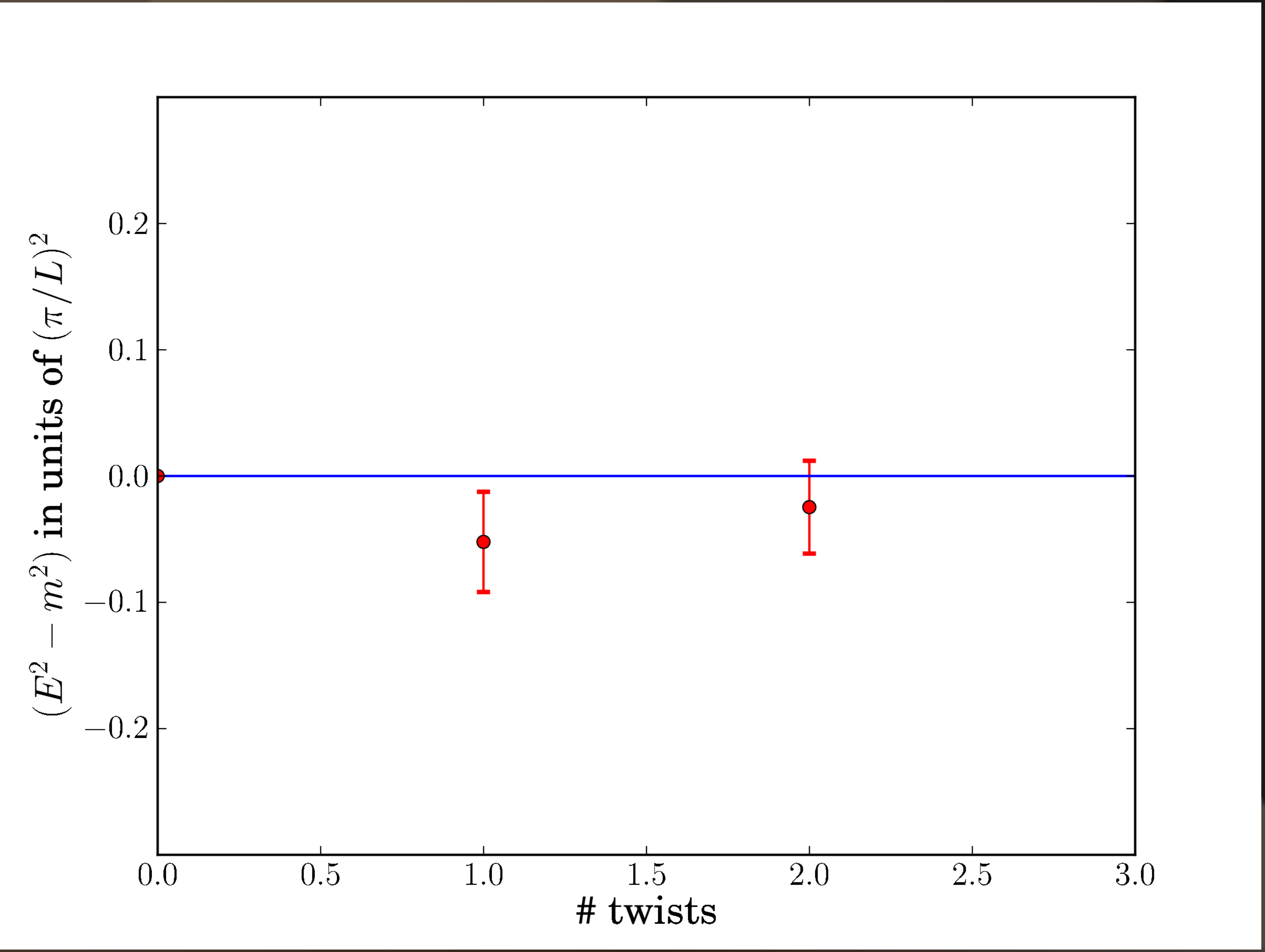
Results: Pion Dispersion Relation

- Generated $16^3 \times 32$ fully dynamical test ensembles with G-parity BCs in 0,1,2 directions.
- $a^{-1} = 1.73(3)\ \mathrm{GeV}$ $m_\pi \sim 420\ \mathrm{MeV}$



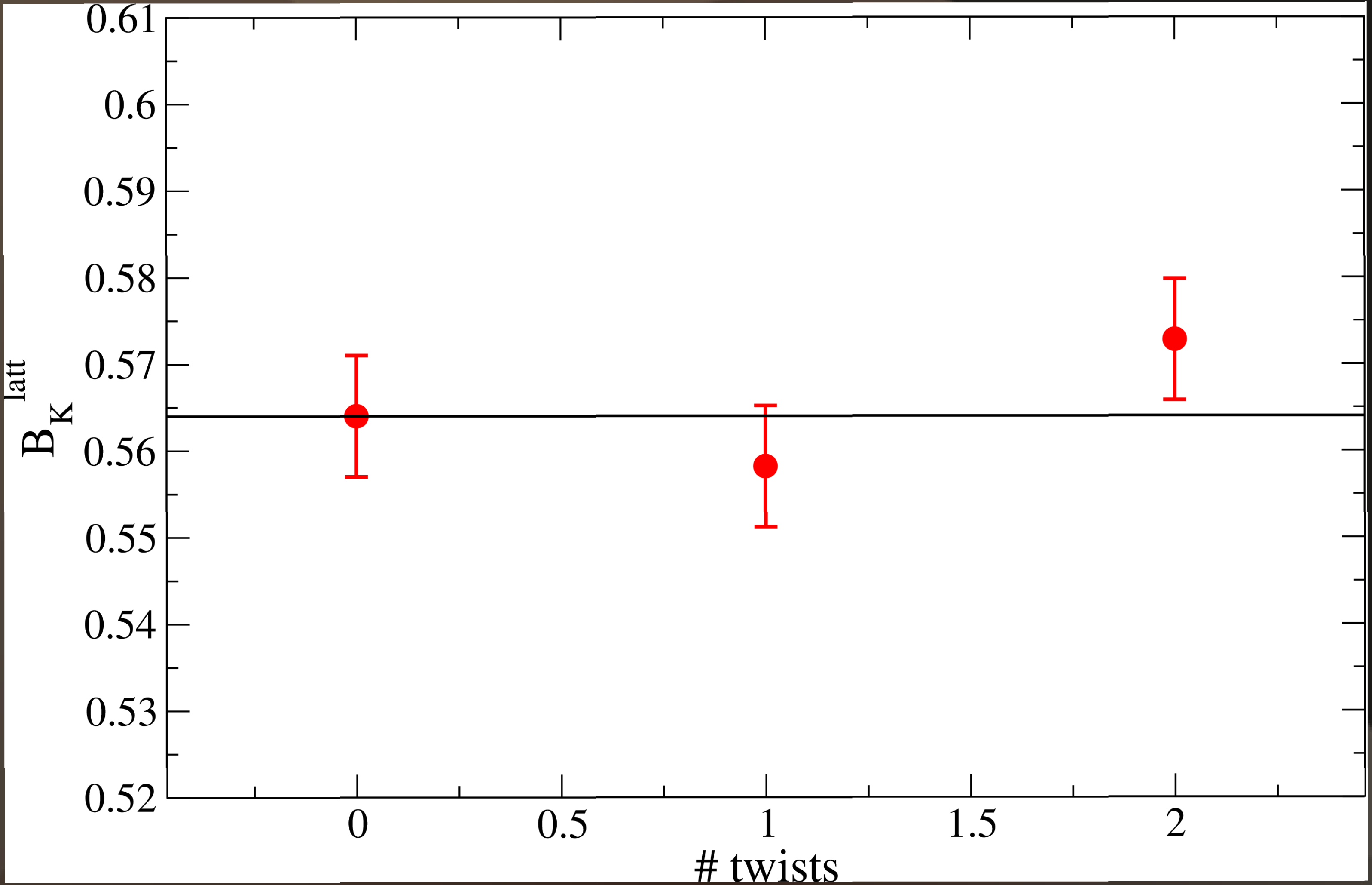
Results: Kaon Dispersion Relation

- Stationary kaon states demonstrated:



Results: B_K

- $\bar{K}^0 \leftrightarrow K^0$ mixing amplitude shown to be independent as expected. These 4-quark effective vertices are similar to those used in $K \rightarrow \pi\pi$ calculation, hence this is a valuable demonstration.



Conclusions and Outlook

Conclusions and Outlook

- Lattice calculations have the potential to lead to great breakthroughs in our understanding of kaon phenomenology, in particular CP-violation.
- In the near future we will begin generating G-parity ensembles with large physical volumes and physical quark masses for a calculation of the $\Delta I = 1/2 \;\; K \to \pi\pi$ amplitude.
- Combining with our existing measurement of the $\Delta I = 3/2$ amplitude will give the first *ab initio* determination of ϵ' . Could potentially lead to discovery of new BSM physics.

Extra Slides

Gauge Field Boundary Conditions

- d-field becomes $C\bar{u}^T$ across the boundary. Consider a bilinear on the boundary under a gauge transformation V:

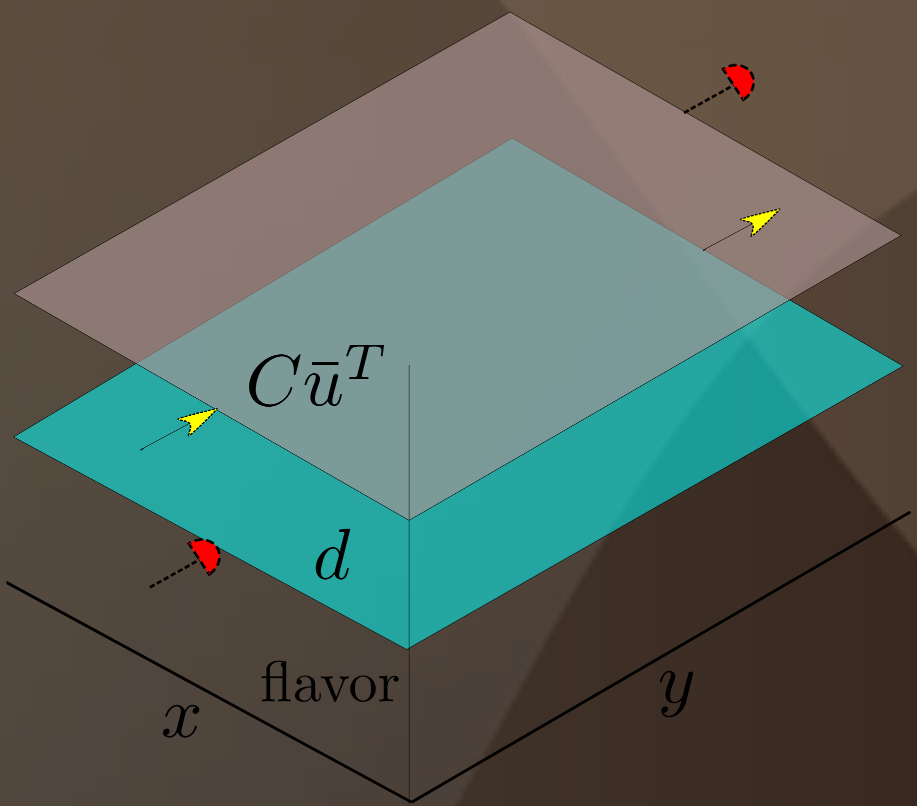
$$\bar{d}(L-1)U_y(L-1)C\bar{u}^T(0)$$
$$\longrightarrow \bar{d}(L-1)V^\dagger(L-1)U_y(L-1)V^*(0)C\bar{u}^T(0)\,.$$

- Link must transform as

$$U_y(L-1) \to V(L-1)U_y(L-1)V^T(0)$$

- Link parallel to boundary on on other side $(y \geq L)$ must then transform as:
- $U_x(x,y,..) \to V^*(x,y,..)U_x(x,y,..)V^T(x+1,y,..)$
- Gauge fields therefore obey complex-conjugate BCs.

The Two-Flavor Method



- Two fermion fields on each site indexed by flavor index:

$$\psi^{(1)}(x) = d(x),\ \psi^{(2)}(x) = C\bar{u}^T(x)$$

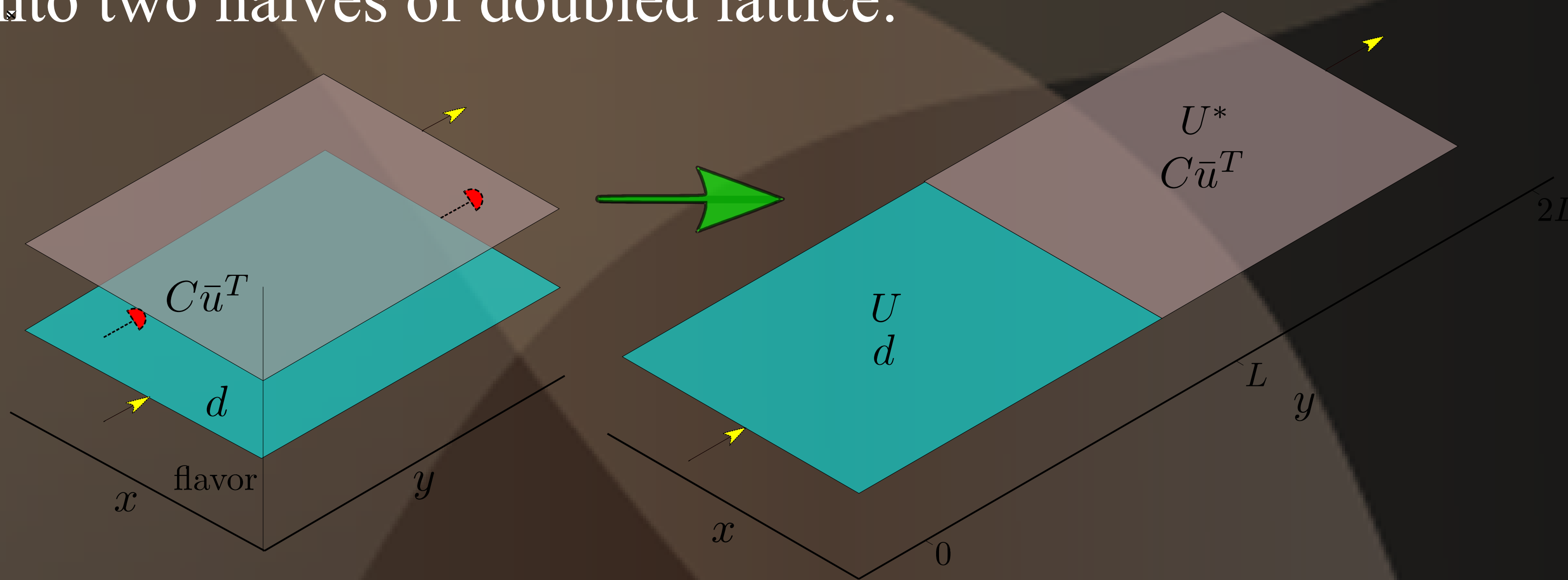
- BCs are:

$$\psi^{(1)}(x + L\hat{y}) = \psi^{(2)}(x),$$
$$\psi^{(2)}(x + L\hat{y}) = -\psi^{(1)}(x),$$

- Periodic BCs in other dirs.
- Single U-field shared by both flavors, with complex conj BCs.
- Dirac op for $\psi^{(2)}$uses U_μ^*.

The One-Flavor Method

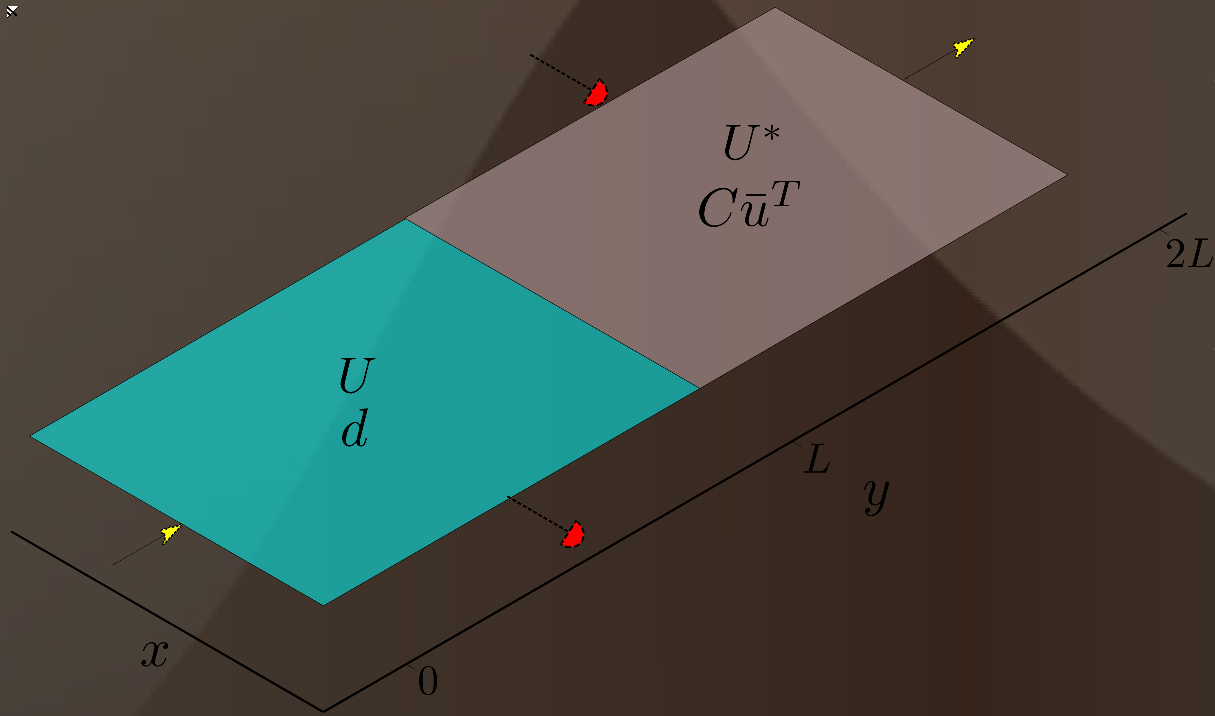
- Obtain equivalent formulation by unwrapping flavor indices onto two halves of doubled lattice:



- Antiperiodic boundary conditions in G-parity direction.
- U -field on first half and U^*-field on second half.

Choosing an Approach

- One flavor setup is much easier to implement.
- However recall that we needed APBC in 2 directions for physical kinematics in $\Delta I = 3/2$ calculation.
- G-parity in >1 dir using one-flavor method requires doubling the lattice again, which is highly inefficient.
- A second approach requires non-nearest neighbour communication:



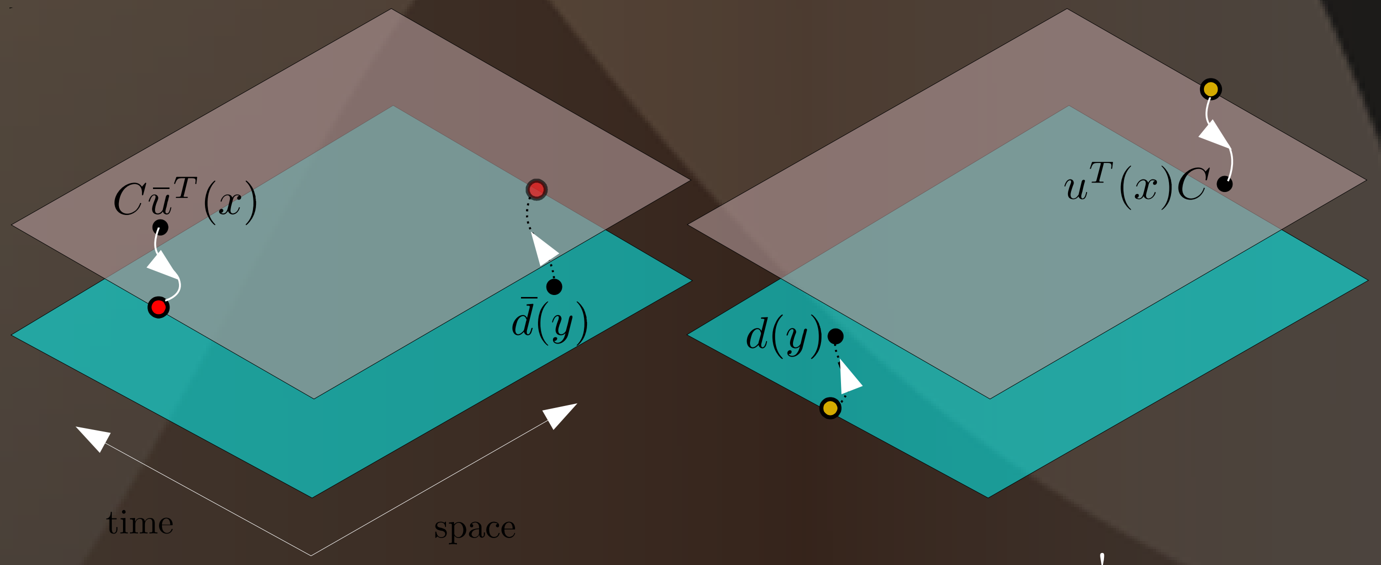
- Also inefficient depending on machine architecture.
- Choose to implement two-flavor method.

Unusual Contractions

- Flavor mixing at boundary allows contraction of up and down fields:

$$\psi^{(2)}_x \bar{\psi}^{(1)}_y = \mathcal{G}^{(2,1)}_{x,y} = C\bar{u}^T_x \bar{d}_y \,,$$
$$\psi^{(1)}_y \bar{\psi}^{(2)}_x = \mathcal{G}^{(1,2)}_{y,x} = -d_y u^T_x C^T$$

- Interpret as boundary creating/destroying flavor (violating baryon number):



- Also have γ^5-hermiticity: $\left[\gamma^5 \mathcal{G}^{(2,1)}_{x,y} \gamma^5\right]^\dagger = \mathcal{G}^{(1,2)}_{y,x}$

Exploiting the Underlying Gauge-Field Symmetry

- Quarks on flavor-1 plane interact with U field, and those on flavor-2 plane with U*.
- Suggests propagators are related in some way.
- In fact, we find that:

$$\begin{aligned} \mathcal{G}^{(2,2)}_{x,z} &= -\gamma^5 C \left[\mathcal{G}^{(1,1)}_{x,z}\right]^* C\gamma^5 \\ \mathcal{G}^{(1,2)}_{x,z} &= +\gamma^5 C \left[\mathcal{G}^{(2,1)}_{x,z}\right]^* C\gamma^5 \end{aligned}$$

- Relative sign due to – sign at boundary between u and d.
- Substantially simplifies contractions.
- In some cases these relations can be used to reduce the number of propagator inversions required.

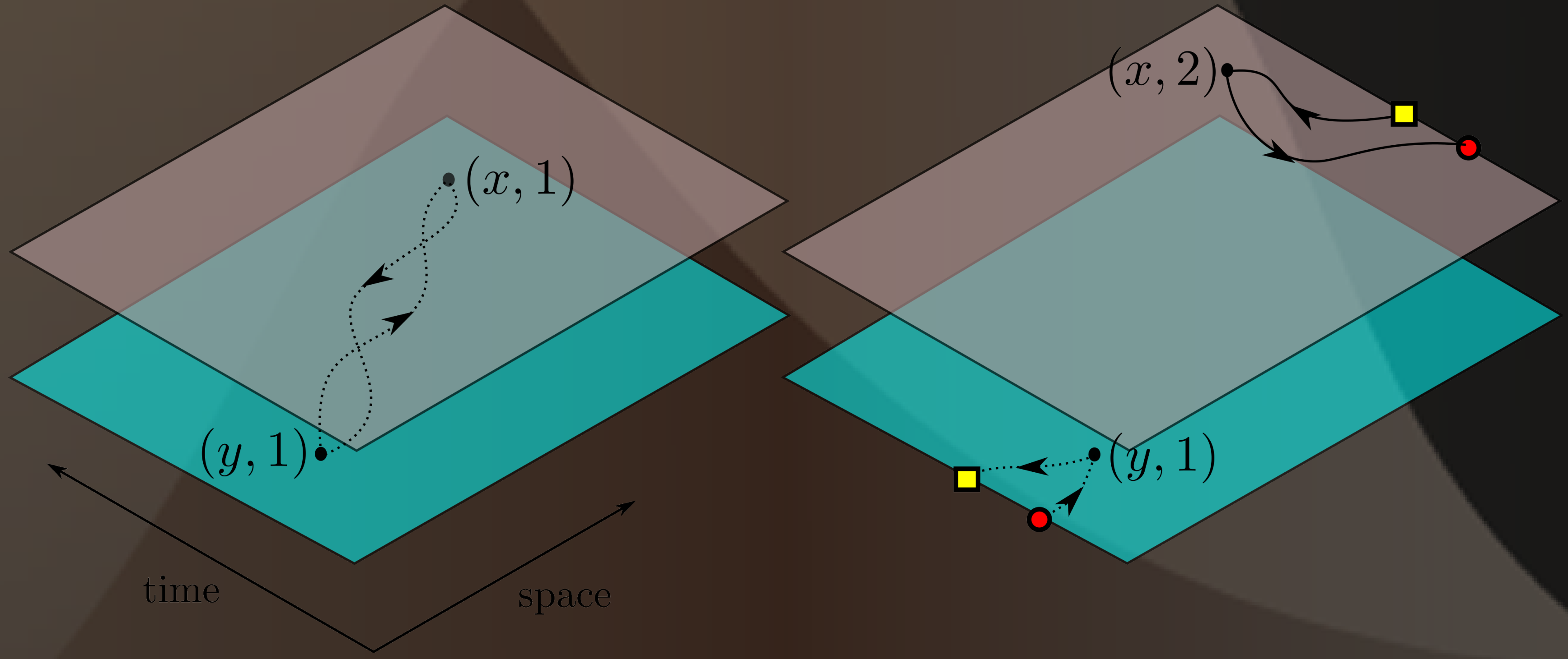
Pion Correlation Functions

- π^+ correlation function

$$\langle \bar{d}_x \gamma^5 u_x \bar{u}_y \gamma^5 d_y \rangle = \langle \bar{\psi}_x^{(1)} [\gamma^5 C] \bar{\psi}_x^{(2)\,T} \psi_y^{(2)\,T} [C \gamma^5] \psi_y^{(1)} \rangle$$

- Now has *two* contractions:

$$\mathrm{tr}\left\{ \mathcal{G}_{x,y}^{(1,1)\,\dagger} \mathcal{G}_{x,y}^{(1,1)} \right\} - \mathrm{tr}\left\{ \mathcal{G}_{x,y}^{(2,1)\,\dagger} \mathcal{G}_{x,y}^{(2,1)} \right\}$$



Locality

- Theory has one too many flavors. Must take square-root of s'/s determinant in evolution to revert to 3 flavors.
- Determinant becomes non-local.
- Non-locality is however only a boundary effect that vanishes as $L \to \infty$. With sufficiently large volumes the effect should be minimal.
- Estimate size of effect?

  - Staggered ChPT?

  - Observe effect of changing from $d \to C\bar{u}^T \to -d$ to $d \to C\bar{u}^T \to +d$ for which $\sqrt{\mathrm{Det}(D)}$ is local (= $\mathrm{Pfaffian}(D)$)?

Charged Kaon Correlator

- K^+analogue: $|K^{+\,\prime}\rangle = \frac{1}{\sqrt{2}}(\bar{u}\gamma^5 s - \bar{s}'\gamma^5 d)|0\rangle$
- 2-point function also has 4 contractions: (flavour indices $3 = s$, $4 = C\bar{s}'^{\,T}$):

$$\frac{1}{2}\mathrm{tr}\left\{\mathcal{G}^{(3,3)\,\dagger}_{x,y}\mathcal{G}^{(1,1)}_{x,y}\right\} + \frac{1}{2}\mathrm{tr}\left\{\mathcal{G}^{(3,3)}_{x,y}\mathcal{G}^{(1,1)\,\dagger}_{x,y}\right\}$$
$$+\frac{1}{2}\mathrm{tr}\left\{\mathcal{G}^{(4,3)\,\dagger}_{x,y}\mathcal{G}^{(2,1)}_{x,y}\right\} + \frac{1}{2}\mathrm{tr}\left\{\mathcal{G}^{(4,3)}_{x,y}\mathcal{G}^{(2,1)\,\dagger}_{x,y}\right\}$$

- If we make the masses of the (s', s) and (u, d) doublets the same this is just the π^+ correlation function but with the *opposite sign* between the contractions.
- Periodicity of spatial dependence appears to arise due to some cancellation between the two contractions.